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# Extreme Nonlin

An Investig

# Elastic Pulse W

# nearity in Rocks:

gation using

# ave Propagation

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# Overview

Our goal in this work is to understand the change of frequency content due to the effect of nonlinear elasticity that takes place during seismic wave propagation. To this end, we have built up an expanded theoretical model describing nonlinear interaction of frequency components of large-amplitude waves in rocks. We also conducted ultrasonic laboratory experiments using 1D wave propagation in rods. Model parameters for first and second nonlinearity parameters $\beta$ and $\delta$ resulting from the pulse-mode investigations agree quite well with estimates from static stress-strain measurements and resonance experiments in the sense that they are several orders of magnitude larger than for ordinary homogeneous materials. Just recently hysteresis effects were included in the model, opening up new perspectives.

# Motivation

Why study NONLINEAR ELASTICITY?
Because this work will lead to:

- New methods by which to characterize rocks and their consolidation and saturation condition.
- Creating a low-frequency seismic source by mixing two high frequency sources.
- Better understanding of seismic wave propagation.
- Better seismic source models (earthquake vs explosion).

# Starting Point
# for Pulse Mode Investigations

**ROCKS ARE HIGHLY DISORDERED MATERIALS:**

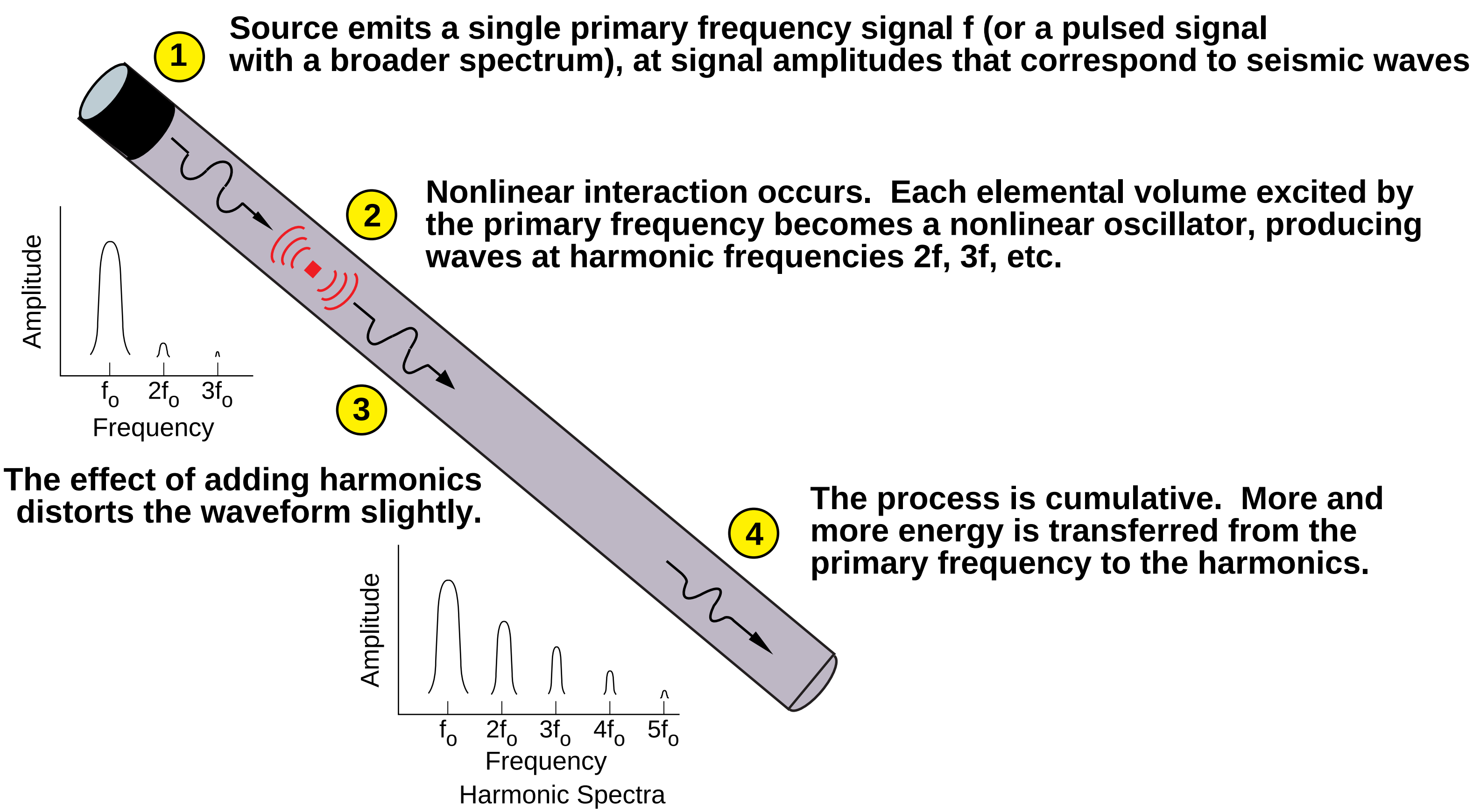
structural defects such as microcracks
& grain-grain boundaries

**PREVIOUS EXPERIMENTS on NONLINEAR ELASTICITY:**

- **STATIC STRESS-STRAIN Measurements**
  - nonlinear and hysteretic (discontinuous)
    - stress-strain/stress-modulus

- **ELASTIC RESONANCE Experiments**
  - harmonic generation (predilection for odd harmonics)
  - resonant peak shift
  - hysteresis

# GOAL: CAN ONE GET CONSISTENT DATA FROM PULSE-MODE INVESTIGATIONS ???



# 1-D Propagation Theory

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x} c_L^2 \left(1 + \beta \frac{\partial u}{\partial x} + \delta \left(\frac{\partial u}{\partial x}\right)^2 + \eta \left(\frac{\partial u}{\partial x}\right)^3 + \xi \left(\frac{\partial u}{\partial x}\right)^4 + \ldots\right)\right] + S_x$$

- **$u$ is the displacement (x distance, t time)**
- **$S$ is the source function**
- $\beta$, $\delta$, $\eta$, $\xi$ **are the first, second, third and fourth nonlinearity parameters**
- **Put** $\beta$, $\delta$, $\eta$, $\xi$ **equal to zero -> Linear case**

  **$C_L$ is the linear wave speed**

# Source Functions

Spectrum of frequencies:

$$S_x(x,\omega) = -2 \, \frac{\partial \, \delta(x)}{\partial \, x} \, 2\pi \sum_{n=-\infty}^{+\infty} U_n \, \delta(\omega - n\omega_0)$$

*with* $U_n = A_n \exp(\, i \, \phi_n \,) = U_{-n}^{cc}$

$\omega_0$ : repetition frequency

Broadband signal:

$$S_x(x,t) = -2 \, i \, k \, \delta(x) \, 2\pi \, \Omega \, t \, e^{-\Omega t}$$

# Solution

Green's function + Perturbation Method:

$$U_n(x+dx) = U_n(x)\,\mathrm{Exp}\left[-\frac{n\,\omega_0}{2\,Q\,c_L}\,dx\right]$$

$$+\frac{\omega_0^2}{2c_L^2}\sum_{m=-\infty}^{+\infty} U_{n-m}(x)\,U_m(x)\,(n-m)\,m\;A(n-m,m)$$

$$+\,i\,\frac{\omega_0^3}{2c_L^3}\sum_{m,l=-\infty}^{+\infty} U_{n-m-l}(x)\,U_m(x)\,U_l(x)\,(n-m-l)\,m\,l\;B(n-m-l,m,l)$$

$$-\frac{\omega_0^4}{2c_L^4}\sum_{m,l,k=-\infty}^{+\infty} U_{n-m-l-k}(x)\,U_m(x)\,U_l(x)\,U_k(x)\,(n-m-l-k)\,m\,l\,k\;C(n-m-l-k,m,l,k)$$

$$-\,i\,\frac{\omega_0^5}{2c_L^5}\sum_{m,l,k,j=-\infty}^{+\infty} U_{n-m-l-k-j}(x)\,U_m(x)\,U_l(x)\,U_k(x)\,U_j(x)\,(n-m-l-k-j)\,m\,l\,k\,j\;D(n-m-l-k-j,m,l,k,j)$$

## Equation for n-th order spectral component

**First term:** **linear propagation with attenuation**

**Other terms:** **nonlinear effects !!!!**

**This equation is a generalization of the Burgers' equation solution for 1D propagation in highly nonlinear solids**

$$A(n-m,m) = \beta \; dx$$

$$B(n,m,l) = \delta \, dx - \beta^2 \, dx \left( \frac{3(n+m+l) - n}{n+m+l} \right) - i \, \frac{(m+l)\,\omega_0}{2\,c_L} \beta^2 \, dx^2$$

$$C(n,m,l,k) = \eta \, dx + \ldots$$

$$D(n,m,l,k,j) = \xi \, dx + \ldots$$

## Remarks:

1. This equation accounts for harmonic generation:
   nonlinear interaction (NI) of frequency components.
   - 1st order NI between $U_{n-m}$ and $U_m$
   - 2nd order NI between $U_{n-m-l}$, $U_m$ and $U_l$
   - ...

2. We use this equation in an iterative procedure:
   - stepwise calculation using small steps dx.
   - use previous output as input for next iteration.

3. Coefficients A, B, C and D differ for different source characteristics:
   distinction between "breathing" (explosive) and "wiggling / sliding" sources (earthquakes).

"Clean" single frequency source of frequency $\omega$ :
(closed circles on next figures)

$2\omega$ harmonic frequency component grows
- linear with propagation distance $x$ from the source
- quadratic with frequency $\omega$
- quadratic with source intensity $A_1$

"Contaminated" multi-frequency source with rep-rate $\omega$ :
(open circles on next figures)

$2\omega$ harmonic frequency component grows CRAZY

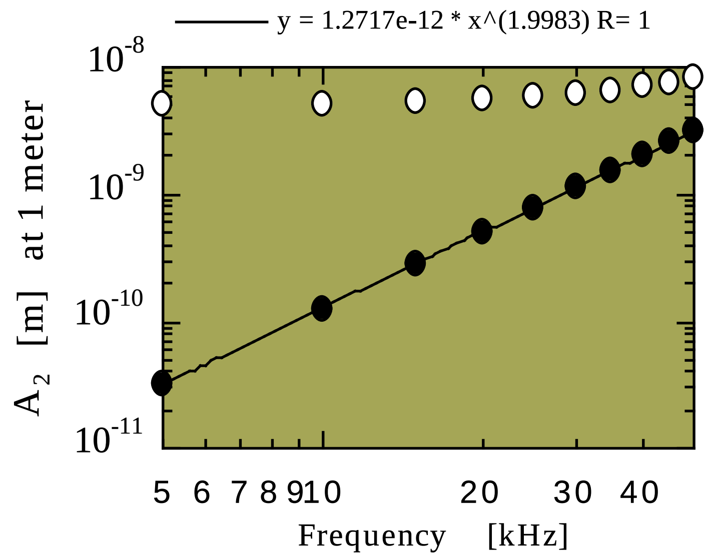
✎ no simple powerlaw dependences ...

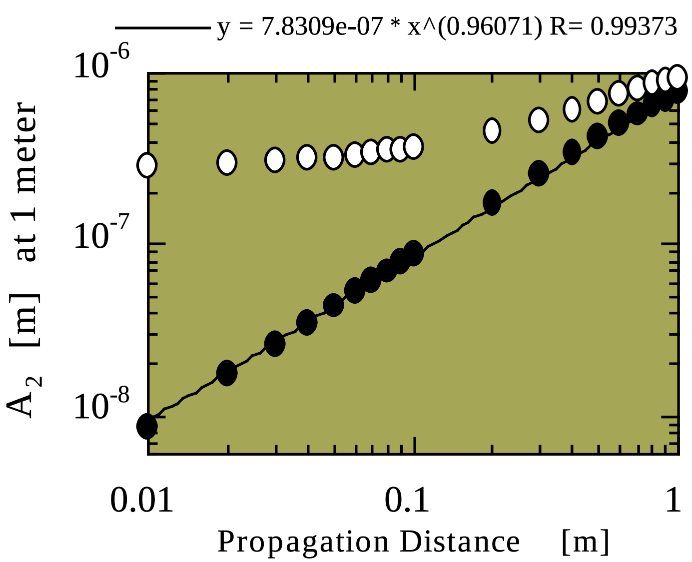
✎ spectral ratios is out of the question ...

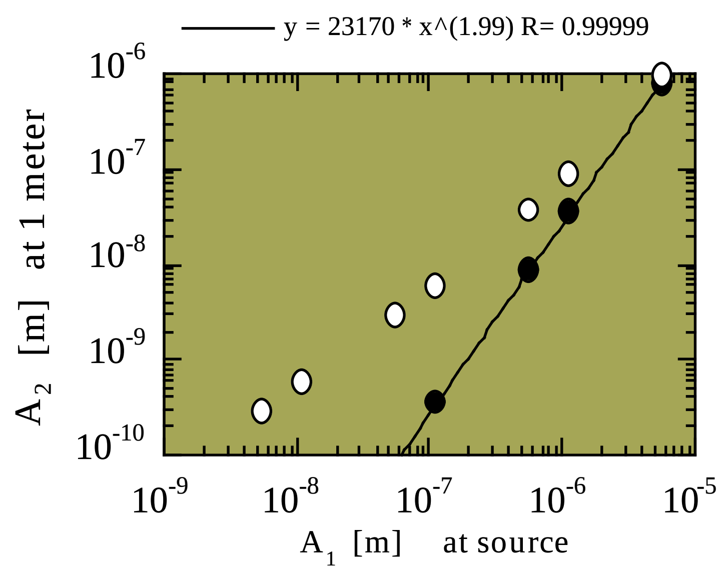
# Second Harmonic Dependences

- ● for a single frequency source

- o for a two-frequency input
  with $A_{2\omega}$ = 5 % of $A_{\omega}$
  at the source



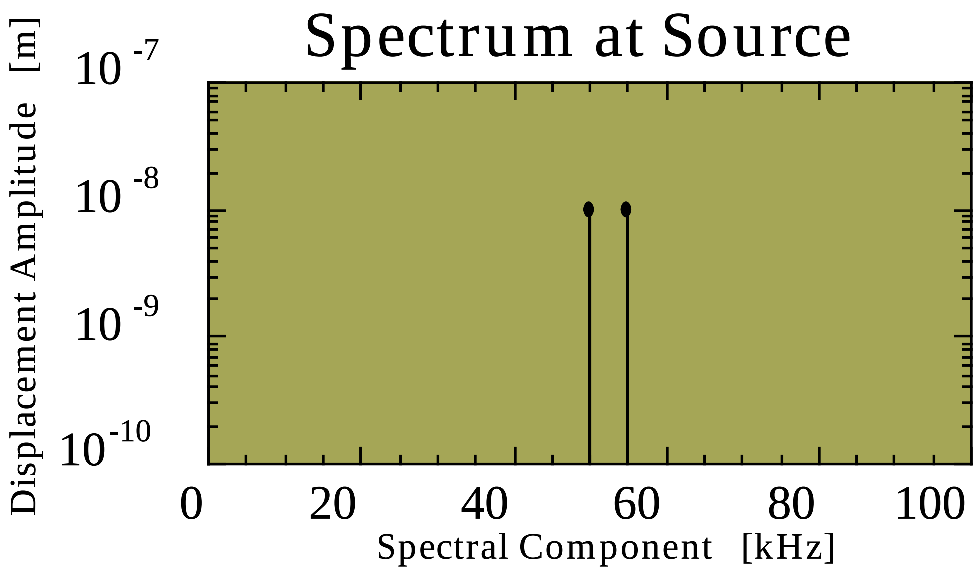



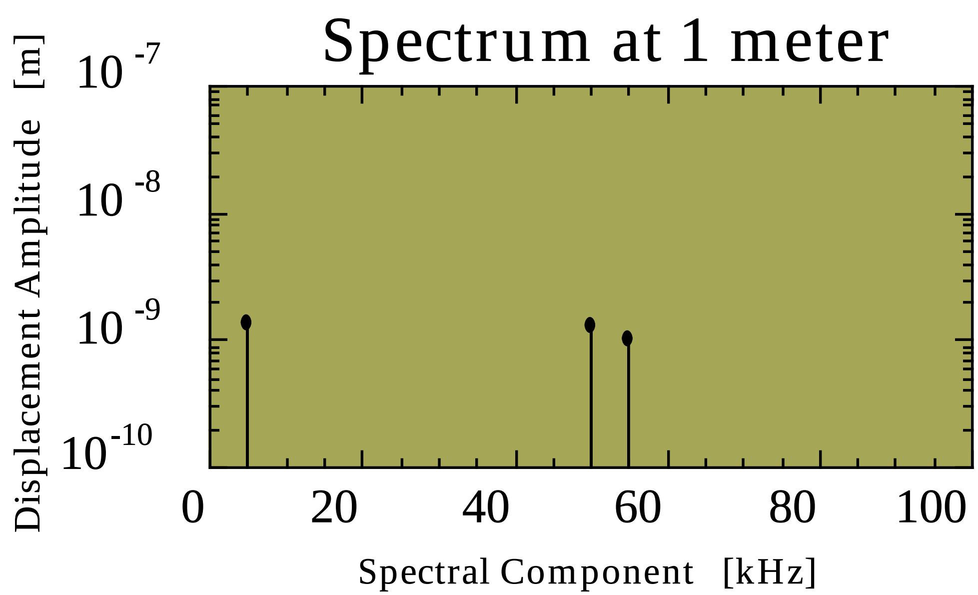



Exploring the possibility
of
PARAMETRIC ARRAYS
in ROCKS

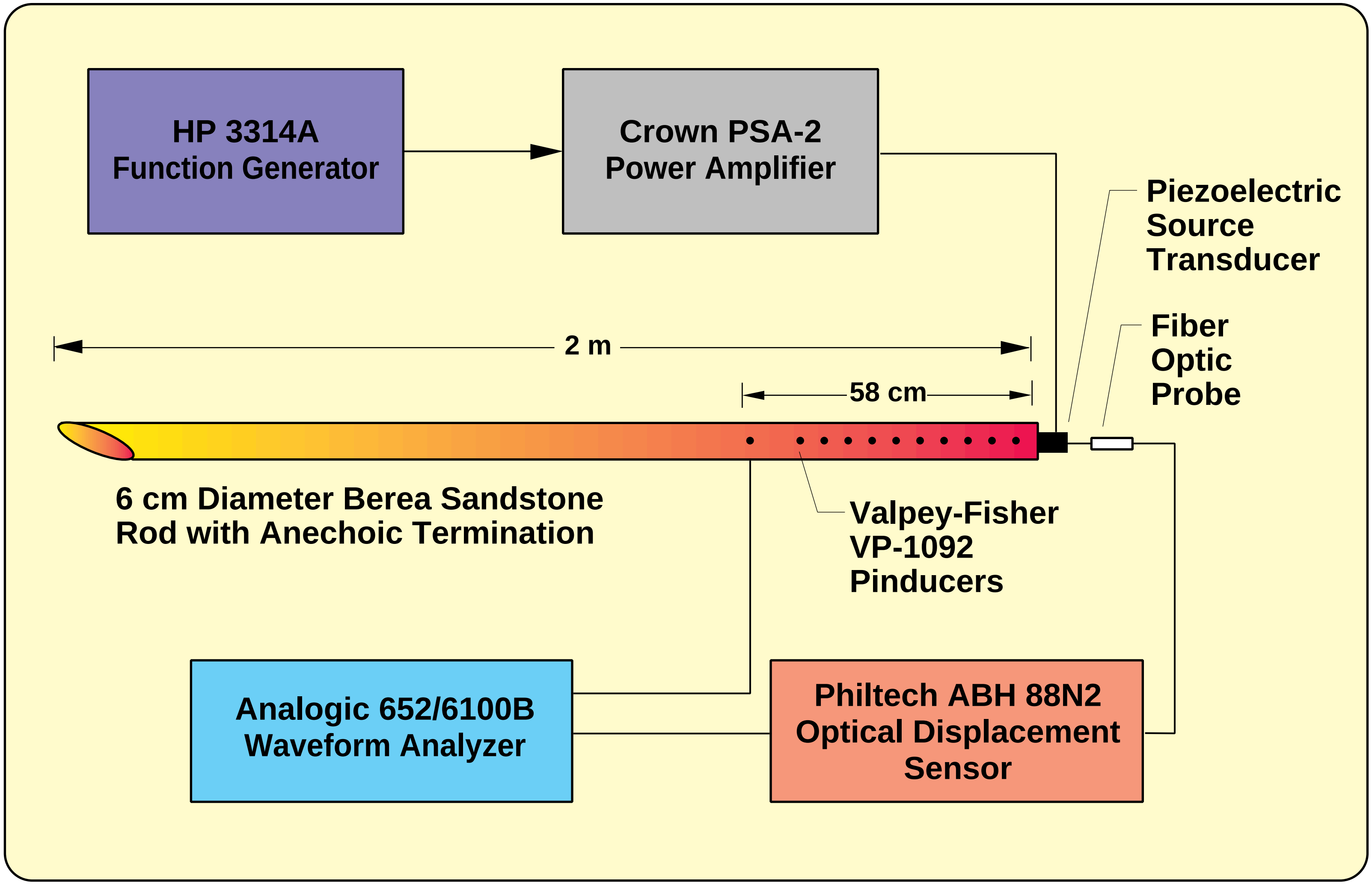
Large nonlinear coefficients

Large Attenuation

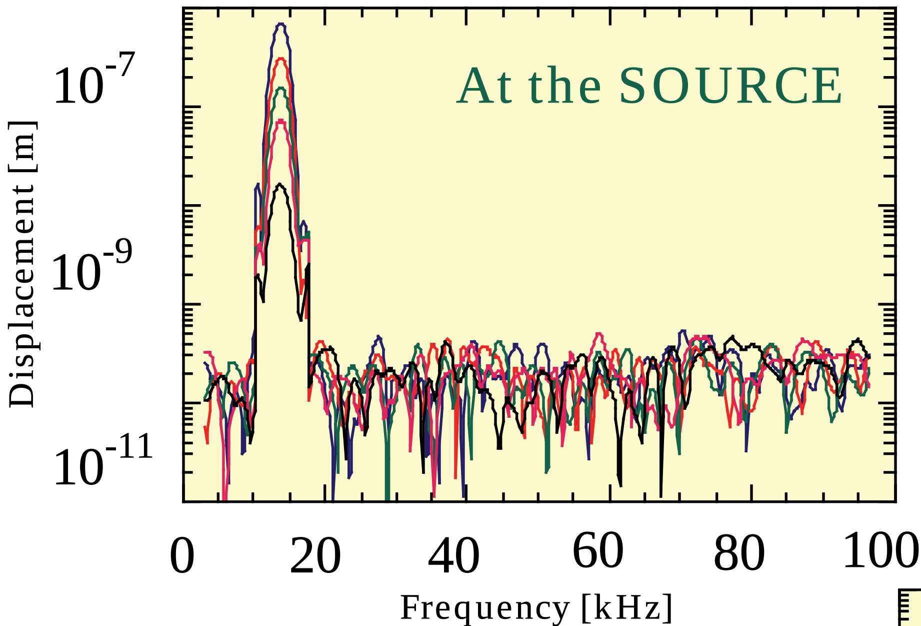


# Experimental Apparatus



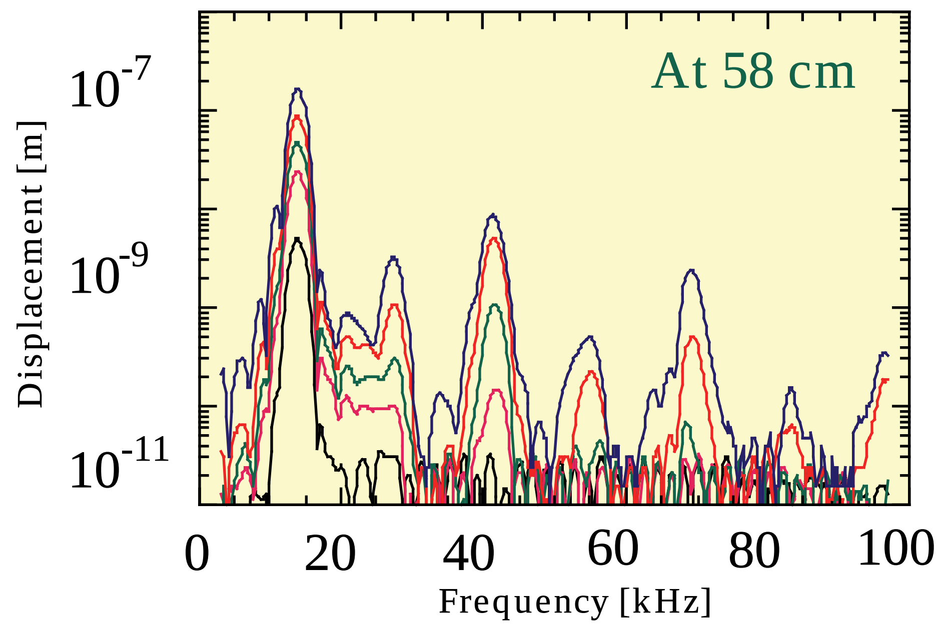

Source displacements ranged from approximately $10^{-9}$Ð$10^{-7}$ m.




Experimental Data
Meegan et al.
(single frequency input)

Displacements for various Intensity levels

Berea Sandstone

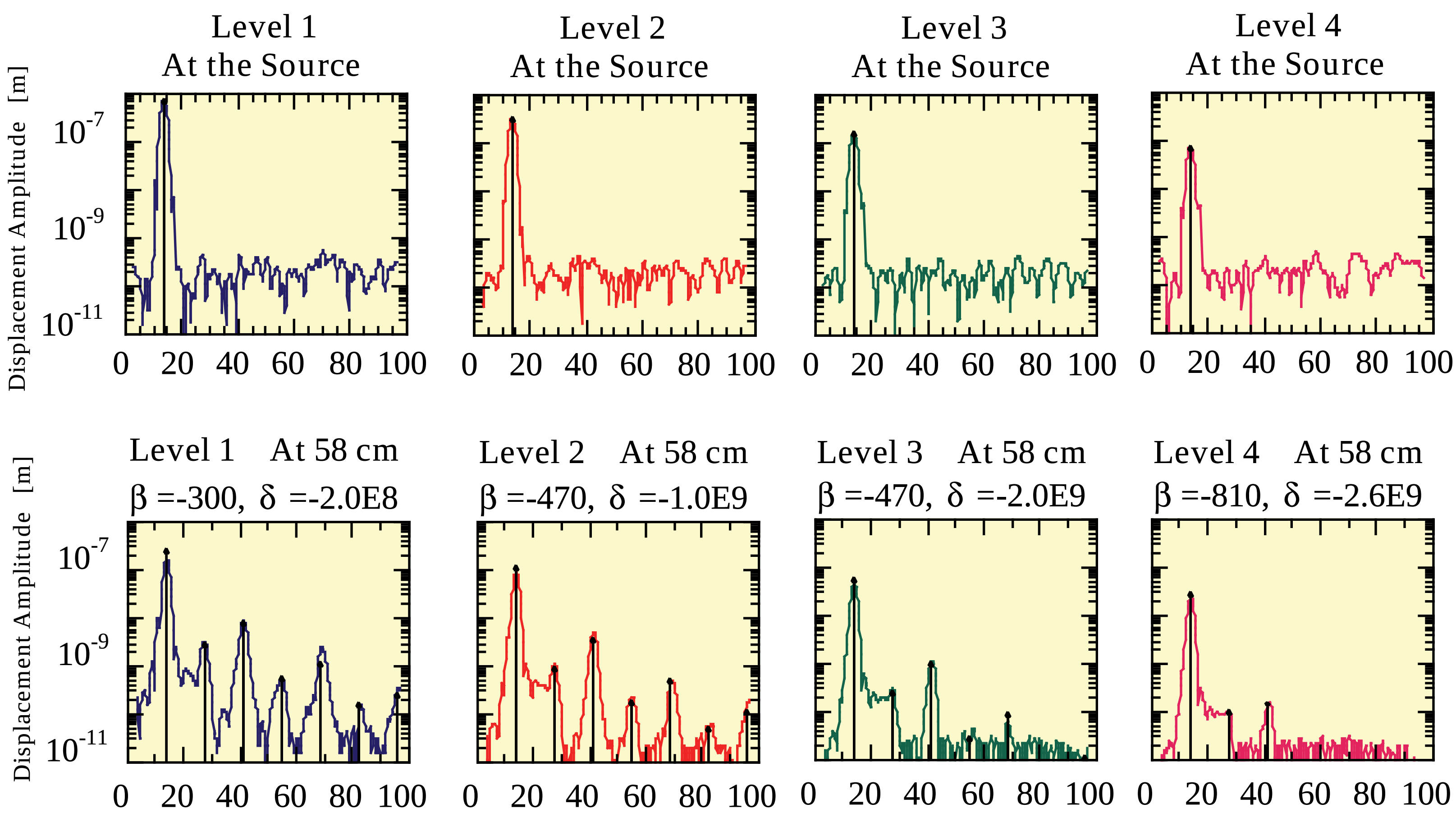


# Comparison Theoretical Model-Experimental Data

Model: bars      Experiment: solid lines

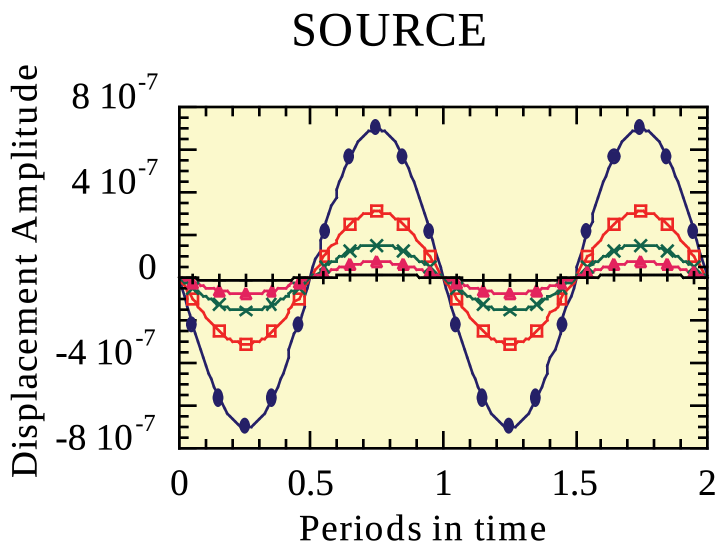
Q = 10, f = 13.75 kHz, c = 2600 m / s

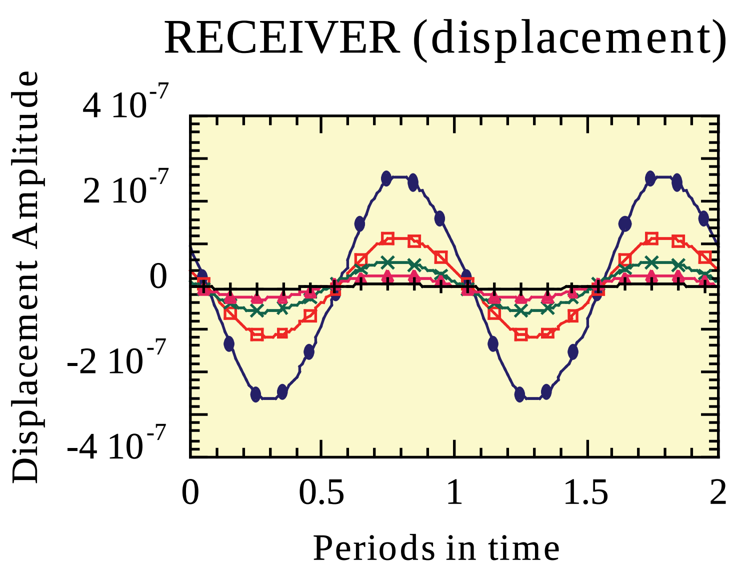
$$\beta \approx -(300 - 810) \quad \delta \approx -(2.0 \cdot 10^{8} - 2.6 \cdot 10^{9})$$

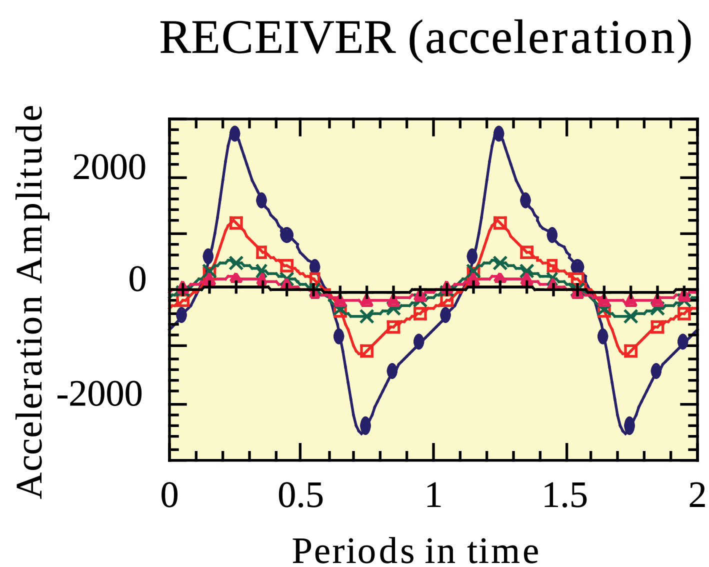


# WAVEFORMS (model results)

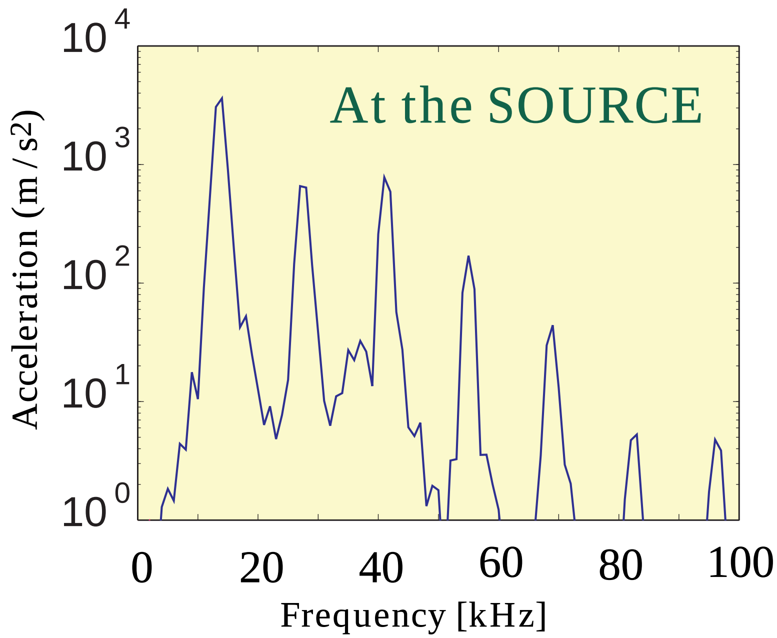
## Source and Receiver (58 cm)



Recently repeated Meegan et al. experiment with an aged, fatigued "messy" source




(multi frequency input)

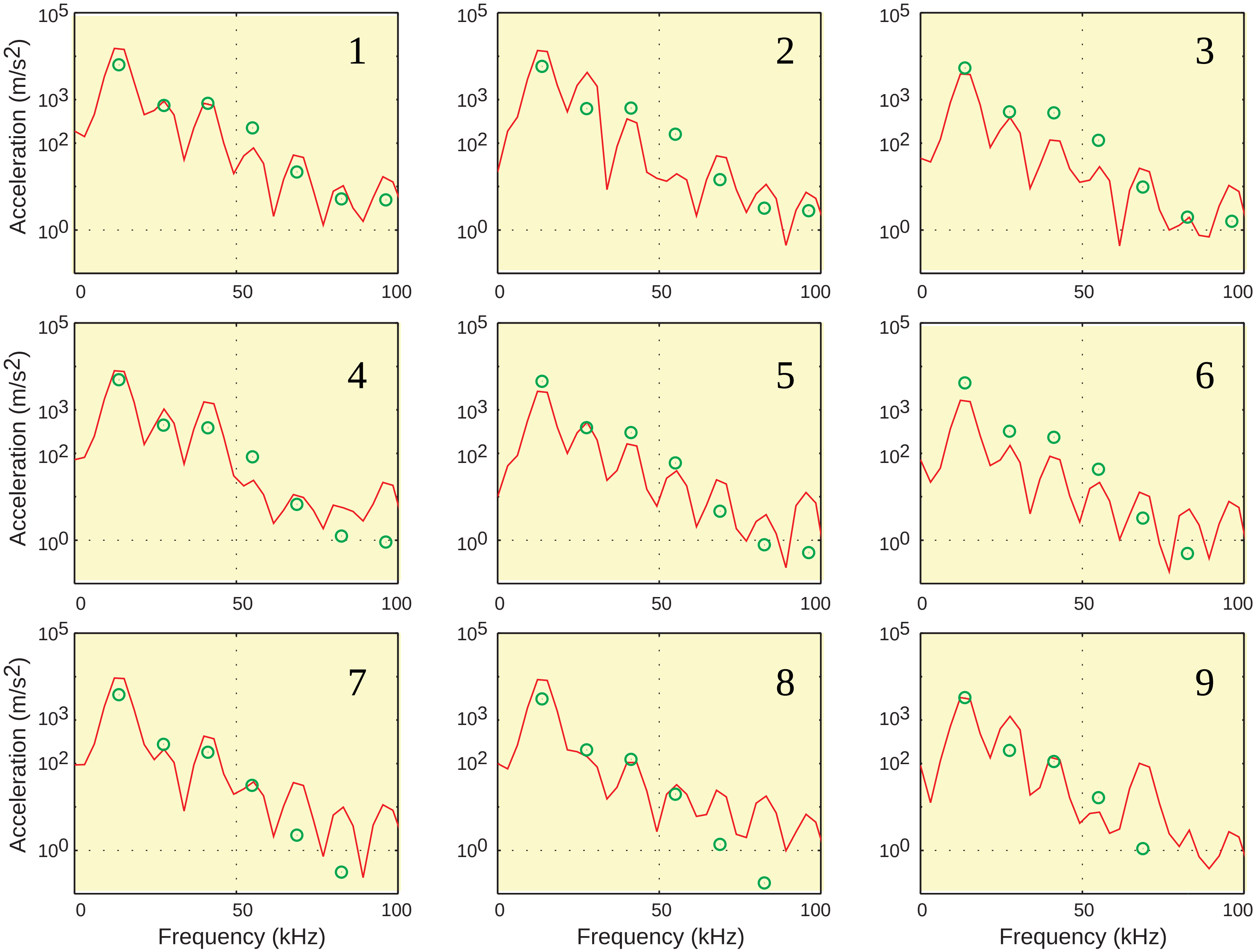
Spectra (shown to the right) generated from the source spectrum shown left were taken at nine distances from the source and are shown in red. Theory predicts the green circles.

Note site response.

Results suggest upper limits for $\beta$, $\delta$ shown.

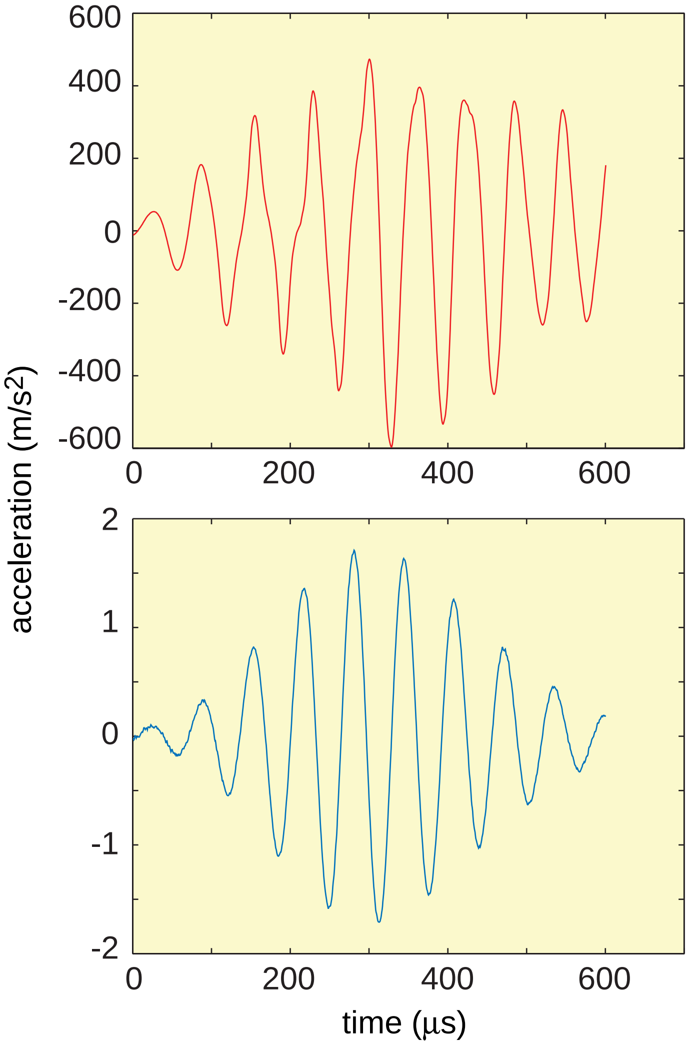
# Comparison Theoretical Model-Experimental Data

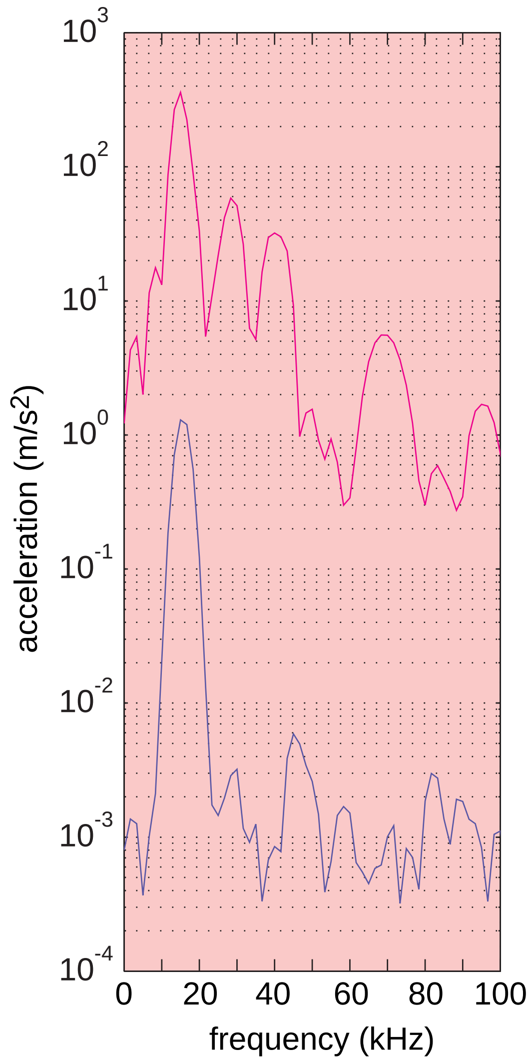
$\beta \leq -(500) \quad \delta \leq -(1.0 \cdot 10^8) \quad Q = 10$



# New experiments in a "thinner" Berea sandstone rod with accelerometers mounted on the surface

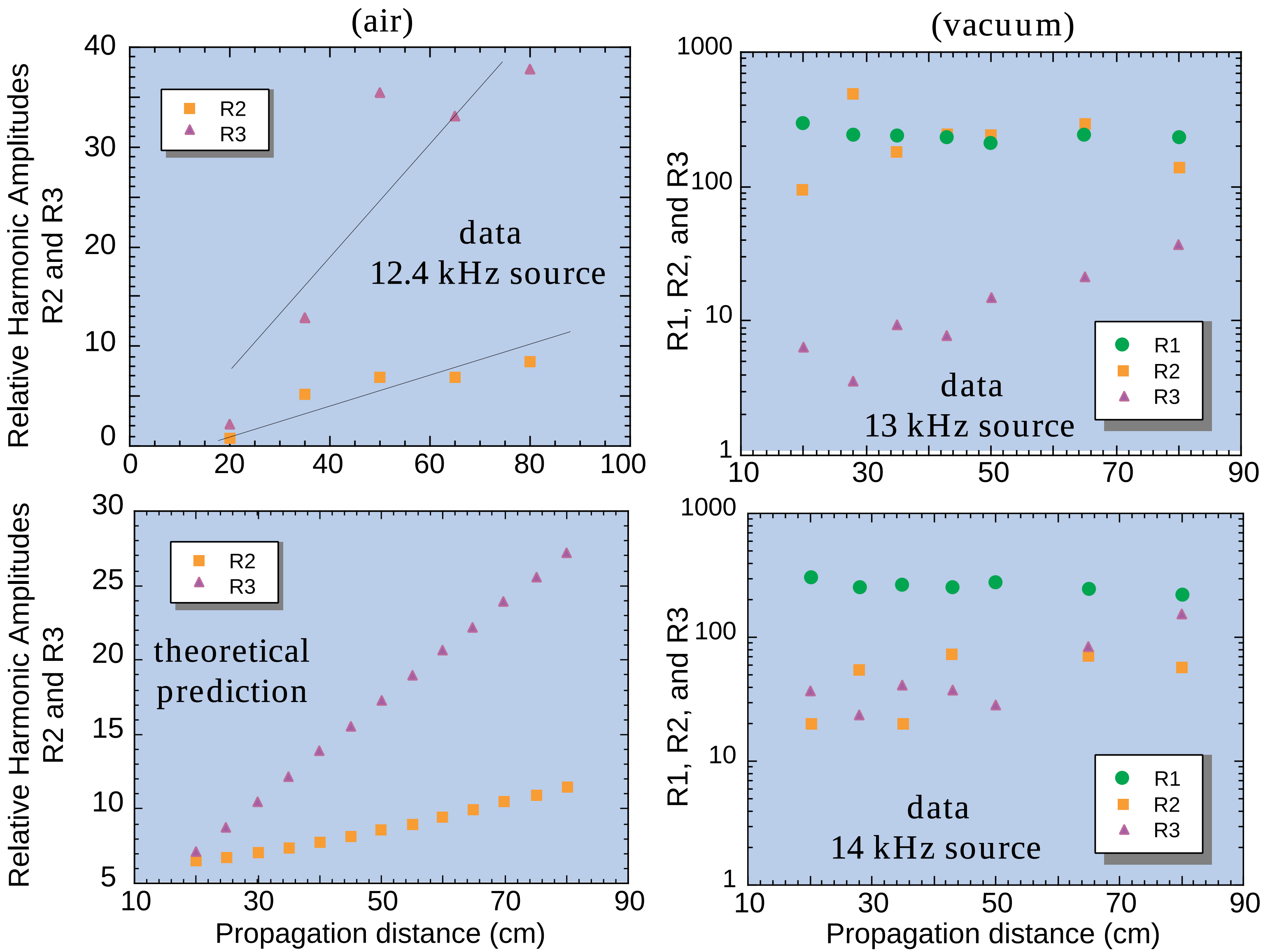
## Linear and nonlinear waveforms and spectra




To account for site response, the method of spectral ratios was utilized. Nonlinearity manifests itself as growth of the spectral ratios of the various harmonics. Plots shown to the right show spectral ratios for experiments in and out of vacuum.

# Spectral Ratios as a function of distance showing harmonic growth of 2nd and 3rd harmonics

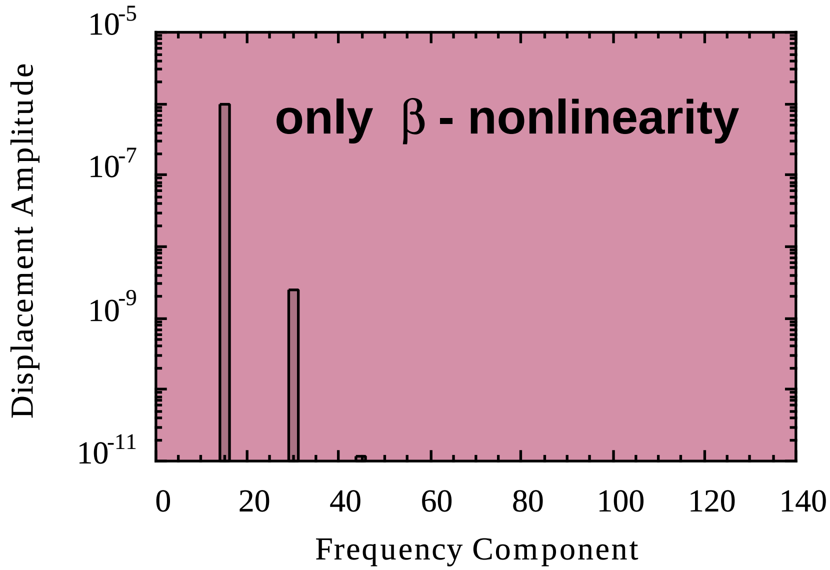


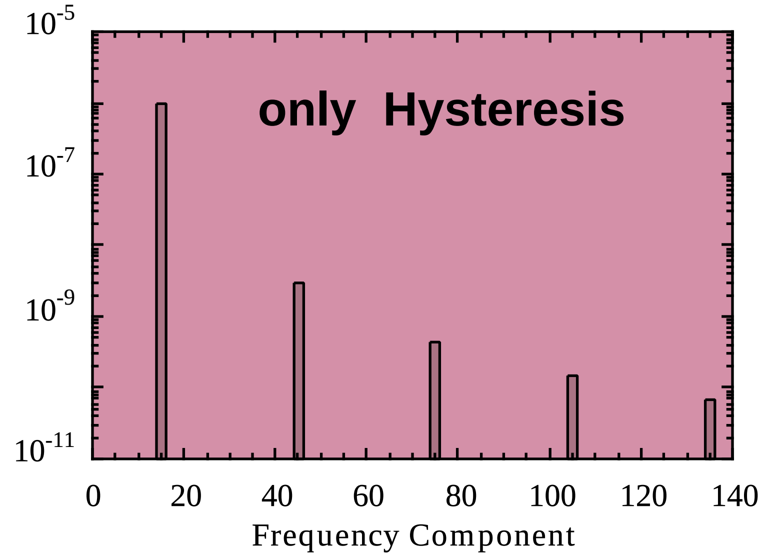

New
New
New
New

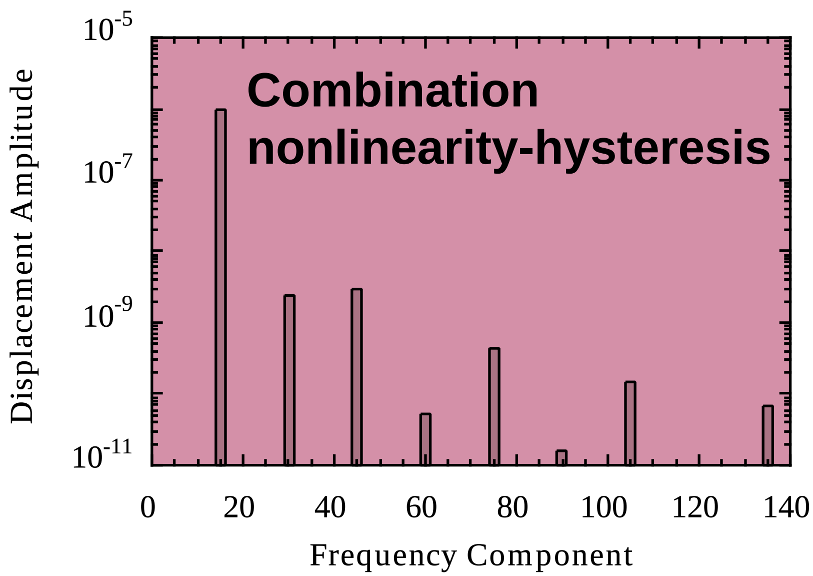
# Hysteretic Model

$$\frac{\partial^2 u}{\partial t^2} = C_L^2 \left[ 1 \pm \alpha(x,t) \frac{\partial u}{\partial x} + \beta \frac{\partial u}{\partial x} + \delta \left( \frac{\partial u}{\partial x} \right)^2 + \dots \right] \frac{\partial^2 u}{\partial x^2} + S_x$$

- $\beta$, $\delta$ **are the first & second nonlinearity parameters**
- $\alpha$ **is the HYSTERETIC TERM that accounts for a discontinuous modulus-stress and a bimodal stress-strain relation**




**Nonlinear wave propagation including hysteretic effects also accounts for predilection of odd harmonics**

# Conclusions of Pulse-mode and Resonant Bar Studies

1. To observe nonlinear response, look at second order effects, e.g., harmonics. If frequency content changes take place in waves generated by natural and explosive sources at seismic frequencies in the earth, then nonlinear elastic effects could eventually influence the way in which seismic explosive sources are modeled.
2. $\beta$ in rock is several orders of magnitude larger than in "linear" materials. Odd harmonics dominate: high $\delta$-term, Intrinsic to (some?) rocks? Consistent magnitude of model parameters $\beta$ and $\delta$ for static tests, resonance and pulse-mode investigations.
3. Hysteresis may be a competitive ingredient in the wave propagation model.
4. This work is leading to a method and theoretical framework for measuring change in saturation, microstructure, density and fatigue in a very sensitive manner. It may well lead to in situ characterization of reservoir rock in boreholes.